

the first topic he looked up (Interpolation, visually-pleasing) had two incorrect page references: there is no material on this topic on pages 106 or 116 (try 112 and 114). However, a random selection over a dozen other index entries revealed no additional errors.

All in all, this reviewer highly recommends *Numerical Methods and Software* to any scientist or engineer who would like insight into current mathematical software or to those who find themselves in the position of teaching a course on numerical mathematics. This reviewer hopes that *he* has the opportunity to teach from this text!

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24[65-02, 65F05, 65F10].—ALEKSANDR A. SAMARSKIĪ & EVGENIĪ S. NIKOLAEV, *Numerical Methods for Grid Equations*, Translated from the Russian by Stephen G. Nash, Vol. I: *Direct Methods*, Birkhäuser, Basel, 1989, xxxv + 242 pp., Vol. II: *Iterative Methods*, Birkhäuser, Basel, 1989, xv + 502 pp., 24 cm. Price \$260.00.

These volumes are devoted to the solution of systems of equations that arise in applying the finite difference method to problems of mathematical physics, mainly to boundary value problems for second-order elliptic equations. They are focused on iterative methods, although direct methods are also discussed. The aim is to gather in one place information on iterative methods for solving difference equations. The book has primarily been written for students of applied mathematics at the Moscow State University. The revised second edition was issued in the Soviet Union in 1987.

There are two volumes. The first volume (Chapters 1-4) deals with the application of direct methods to the solution of difference equations, the second volume (Chapters 5-15) considers iterative methods.

Chapter 1 provides the necessary foundations for solving linear difference equations. Chapter 2 describes some variants of the Gauss elimination method for solving one-dimensional 3- and 5-point difference equations. In Chapter 3 the cyclic reduction method is studied, and Chapter 4 deals with the separation of variables method (FFT method) for solving Poisson's difference equations in a rectangle.

The theory of iterative methods is introduced in Chapter 5. Iterative methods are considered as operator difference schemes. This approach has many advantages. The method does not depend on a choice of particular basis functions and on a representation of the operators in this basis. The authors introduce the

concept of a *canonical form* of an iterative method. It is possible to compare two different iterative methods by comparing their canonical forms. Studying the convergence rate of an iterative method, it is sufficient to know only some properties of the difference scheme operators such as symmetry, positive definiteness and lower and upper estimates of the spectrum of the operators. Necessary concepts from functional analysis are provided.

Chapters 6 and 7 discuss two- and three-level iterative methods (with Chebyshev parameters). Iterative methods of variational type (also the conjugate gradient method) and Gauss-Seidel and SOR methods are the subject of Chapters 8 and 9. Chapter 10 deals with the alternate triangular method. The authors consider it as a fundamental method in the book. The method was very effective for solving the Dirichlet problems in an arbitrary region at that time (up to 1980), but nowadays more effective methods are known (mainly the domain decomposition or multigrid methods). The next five chapters investigate the alternating direction (ADI) methods, iterative methods for non-self-adjoint equations with indefinite and singular operators, iterative methods for solving nonlinear difference equations and for solving elliptic equations in curvilinear coordinates. In Chapter 14 example solutions of elliptic grid equations are given (multi-dimensional problems, schemes for equations of elasticity theory, etc.).

The mathematical level of the text is very high. Apart from some introductory lemmas and theorems in Chapters 1 and 5, every theorem and lemma has a proof. The book is directed at advanced readers. However, it contains many remarks and examples illustrating the methods discussed. (Poisson's equation on a square region with the Dirichlet boundary conditions is a model problem which is intensively used in comparing various methods.) The methods considered are investigated from a mathematical point of view. For iterative methods the convergence rate and the choice of optimal parameters, for which the convergence rate is maximal, are studied. Estimates are given for the number of iterations required. The implementation aspects of the methods are briefly discussed. Some computational comparisons of the methods are given for the model problem.

The book deals only with difference scheme equations. The finite element method is not touched upon, although it is a very active area of research. Nevertheless, the book presents the broad picture of iterative methods and contains a large number of iterative methods with their detailed analyses. It is recommended for those acquainted with fundamentals of functional analysis and finite difference methods. The book is a classic, and should be a valuable addition for practitioners as well as students in the field.

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